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The influence of Coulomb repulsion and fluctuation effects on the critical temperature in layered superconductors

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Abstract. The influence of the electronic Coulomb repulsion on the critical temperature for a layered superconductor is studied by using the Eliashberg theory. The dependence of the critical temperature, T_c, on the interlayer distances, tunnelling integral $t_\perp$ and the thickness of the conducting layers is established.

The Lawrence–Doniach free energy functional for quasi-two-dimensional superconductors is used to study the influence of fluctuations of the order parameter phase on T_c and the dependence of T_c on $t_\perp$ is also obtained. The critical temperature appears to increase with $t_\perp$ for T_c within the range $T_c^* < T_c < T_{c0}^{(2)}$, where $T_{c0}^{(2)}$ is the critical temperature formally evaluated by means of BCS theory and $T_c^* = (1/T_{c0}^{(2)} + 1/2\pi\epsilon_F)^{-1}$.

1. Introduction

Discovery of a new class of high-temperature superconductors containing thallium ($Tl_mBa_2Ca_{n-1}Cu_nO_{2(n+1)+m}$) and bismuth ($Bi_2Sr_2Ca_{n-1}Cu_nO_{2n+4}$) [1–3] has confirmed once again that variation of the structure anisotropy strongly affects the transition temperature T_c, upper and lower critical fields, coherence length and other important parameters of superconductors (SCs). In these materials the number n of superconducting CuO_2 planes per unit cell, as well as the distance between the planes are changed with variation of calcium concentration.

The recent developments in epitaxial technology allow the growth of artificial SC/dielectric superlattices [4–6] in which the thickness of planes varies within a wide range. Variation of superlattice parameters such as thickness and materials of layers strongly influences the critical temperature T_c. Therefore, finding the dependence of T_c on the thickness of SC and dielectric layers, and also on the tunnelling integral between SC layers, becomes an interesting problem.

In the present paper, strongly anisotropic SCs are studied on the basis of the Lawrence–Doniach model [6, 7]. In this model two-dimensional (2D) SC planes

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weakly interact with each other by Josephson coupling. Here the energy spectrum of electrons is supposed to have the form

$$\xi(\boldsymbol{K}, K_z) = (K^2/2m) + t_\perp[1 - \cos K_z(a+d)] \tag{1}$$

where $\boldsymbol{K}$ is the 2D impulse vector inside a conducting layer, K_z is the z component of the impulse vector $(\boldsymbol{K}, K_z)$, $t_\perp$ is the interlayer tunnelling integral, d is the SC layer thickness and a is the distance between these layers (for superconducting superlattices d and a are the SC and dielectric layer thicknesses, respectively). $t_\perp$ characterizes the intensity of electron tunnelling between the layers and must depend on the ratio a/b as $t_\perp = F(a/b)$, where b is a characteristic distance of the order of unit cell size in the SC layer. The function $F(a/b)$ rapidly decreases when the distance a increases. It is possible, in principle, to obtain an explicit expression for this function provided that the electron density distribution inside the SC layers is known.

A strongly anisotropic Fermi surface corresponds to the case of $t_\perp \ll \epsilon_F = p_F^2/2m$, where ϵ_F is the Fermi energy. Then, the Fermi surface becomes open as a corrugated cylinder.

In the absence of Josephson connection between the layers, fluctuations of the order parameter phase would destroy the long-range order [9,10]. Nevertheless, the existence of topological defects in a 2D SC, such as 'vortices' and 'antivortices' of a phase field, should result in the Berezinskii–Kosterlitz–Thouless phase transition at $T = T_{KT}$ [11–13]. The same defects give rise to a quasi-long-range order at $T_{KT} < T < T_{c0}^{(2)}$, where $T_{c0}^{(2)}$ is a critical temperature formally evaluated by the mean field theory for a single SC plane.

It has been shown [14,15] that at sufficiently low value of Josephson coupling constant, $W_\perp = t_\perp^2/\epsilon_F$, weak interaction of the vortices at different SC planes may bind together the Kosterlitz–Thouless vortices as vortex loops. Therefore, the dependence of the critical temperature T_c on $W_\perp$ can be expressed as

$$T_c = T_{KT}[1 + 1/\ln^2(\hbar^2 N_s/2mW_\perp)] \tag{2}$$

where N_s is the surface density of electrons.

Friedel has shown [15,16] that plane dislocation loops whose core is located between the SC layers can be created in quasi-2D superconductors. These defects, according to Friedel, are created more easily and lead to a dependence of $\tilde{T}_c$ on $W_\perp$ of the form

$$\tilde{T}_c = T_{c0}^{(2)}/[1 + \ln(\epsilon_F/W_\perp)]. \tag{3}$$

In this paper we study the dependence of the critical temperature T_c on the interlayer tunnelling integral $t_\perp$, the distance a between SC layers, and also the thickness d of SC layers, caused by Coulomb interaction between electrons. In this connection we take the thickness d of SC layers to vary in an interval which satisfies the condition $(a+d) < \xi_\perp$ [17] where $\xi_\perp$ is the correlation length perpendicular to the layers.

Further, starting from the Lawrence–Doniach free energy functional [6,8] we find the dependence of T_c on $W_\perp$ caused by the classical fluctuations of the order parameter phase.

2. Coulomb repulsion effects in layered superconductors

We shall use the Eliashberg theory [18,19] to obtain the transition temperature. This method has been applied to layered systems by other authors [20,21] with the purpose of studying their superconducting properties. Therefore, without detailed calculations we shall start from the following gap equation for $\Delta(x, y)$ in a coordinate representation

$$\Delta(x,x') = \int \mathrm{d}^4y\,\mathrm{d}^4z\; G(x,y)G(x',z)\Delta(y,z)\Gamma(y,z) \tag{4}$$

where $x = \{r,\omega\}$; $G(r,r';\omega)$ is an electron Green function and $\Gamma(r,r';\omega)$ is a kernel of the Eliashberg equation. In a momentum representation the kernel $\Gamma(p,\omega)$ is represented as $\Gamma(p,\omega) = D(p,\omega)+\tilde{U}(p,\omega)$ which implies the contributions from the electron–phonon interaction, $D(p,\omega)$, and Coulomb interaction, $\tilde{U}(p,\omega)$.

In a weak-coupling limit where electron–phonon coupling constant λ is smaller than unity, i.e. $\lambda \ll 1$, the Eliashberg equation gives an analytical solution and the well known Bardeen–Cooper–Schrieffer (BCS) expression for T_c as

$$T_c = (2\gamma/\pi)\omega_D \exp[-1/(\lambda-\mu^*)] \tag{5}$$

where ω_D is the Debye temperature, μ^* is the Coulomb pseudopotential and $\gamma = \mathrm{e}^c = 1.78\ldots$, the Euler–Mascheroni constant.

Different frequency scales of λ and μ lead to weakening of μ as

$$\mu^* = \mu/[1+\mu\,\ln(\epsilon_F/\omega_D)] \tag{6}$$

where μ is the screened Coulomb interaction averaged over the Fermi surface. Averaging of the screened Coulomb potential $\tilde{V}(K,K_z)$ over the Fermi surface is performed according to the formula

$$\mu = \frac{D}{2\pi}\int_{-\pi/D}^{\pi/D}\mathrm{d}z\frac{2\nu(0)}{\pi}\int_0^{2P_F}\frac{\mathrm{d}K}{[(2P_F)^2-K^2]^{1/2}}\tilde{V}(K,K_z) \tag{7}$$

where $\nu(0) = m/\pi\hbar^2$ is the 2D density of states on the Fermi surface and $D = a+d$. To calculate μ, we shall use the expression for the bare Coulomb interaction $V(K,K_z)$ [22] of charged particles in a metal–dielectric superlattice:

$$V(K,K_z) = \frac{2\pi e^2}{\epsilon K}\frac{\beta_1\sinh K(a+d)+\beta_2\sinh K(a-d)}{\beta_1^2\cosh K(a+d)-\beta_2^2\cosh K(a-d)-\eta\cos(a+d)K_z} \tag{8}$$

where K is a modulus of the 2D impulse vector inside the conducting layer, d is the thickness of the conducting layer and a is the distance between them (or the dielectric layer thickness in artificial metal/dielectric superlattices).

In (8), the following definitions are introduced: $\beta_1 = \frac{1}{2}(1+\eta)$, $\beta_2 = \frac{1}{2}(1-\eta)$, where $\eta = \epsilon_1/\epsilon$; with ϵ and ϵ_1 being the static permittivities of a metal and dielectric, respectively.

Divergence of the bare potential $V(\boldsymbol{K}, K_z)$ at small impulses may be removed by summing the loop diagrams [19,23]. Using the Dyson equation the screened Coulomb potential $\tilde{V}(\boldsymbol{K}, K_z; \omega)$ is expressed by an electron polarizator $\Pi(\boldsymbol{K}, K_z; \omega)$ as

$$\tilde{V}(\boldsymbol{K}, K_z; \omega) = V(\boldsymbol{K}, K_z)/[1 + V(\boldsymbol{K}, K_z)\Pi(\boldsymbol{K}, K_z; \omega)]. \tag{9}$$

The polarization operator $\Pi(\boldsymbol{K}, K_z; \omega)$ is calculated by the following expression:

$$\Pi(\boldsymbol{K}, K_z; \omega) = 2 \sum_{\boldsymbol{P}, P_z} \frac{n[\xi(\boldsymbol{P}+\boldsymbol{K}, P_z + K_z; t_\perp)] - n[\xi(\boldsymbol{P}, P_z; t_\perp)]}{\xi(\boldsymbol{P}, P_z; t_\perp) - \xi(\boldsymbol{P}+\boldsymbol{K}, P_z + K_z; t_\perp) + \mathrm{i}\omega} \tag{10}$$

where n is the Fermi distribution. The electron energy spectrum ξ is defined by equation (1). At zero frequency, we integrate over $\boldsymbol{P}$ and obtain

$$\frac{\Pi(\boldsymbol{K}, K_z; \omega)}{\Pi(0)} = 1 - \frac{1}{K^2} \int_{-\pi/D}^{\pi/D} \frac{\mathrm{d}P_z}{2\pi} \sqrt{A^2 - (2KP^*)^2}\, \vartheta[A^2 - (2KP^*)^2] \operatorname{sgn} A \tag{11}$$

where

$$\Pi(0) = m/\pi\hbar^2 \qquad A = K^2 + 4mt_\perp \sin(K_z D/2) \sin(p_z D)$$

and

$$P^* = \{p_F^2 - 2mt_\perp[1 - \cos(p_z - K_z/2)D]\}^{1/2}.$$

In (11), $\vartheta(x)$ is a step function, i.e. $\vartheta(x) = 1$ for $x > 0$; $\vartheta(x) = 0$ for $x < 0$, and $\operatorname{sgn}(x) = 1$ for $x > 0$; $\operatorname{sgn}(x) = -1$ for $x < 0$. For strongly anisotropic systems, where the condition $t_\perp/\epsilon_F \ll 1$ is satisfied, we can also integrate (11) over p_z. As a result we obtain $\Pi(K, K; 0)$ as

$$\frac{\Pi(\boldsymbol{K}, K_z; 0)}{\Pi(0)} = 1 - \frac{1}{\pi K} \begin{cases} 4\sqrt{K^2 - 4p_F^2 + 8mt_\perp} \\ \quad \text{for } K^2 > 4p_F^2 - 8mt_\perp(1 - |\cos\tfrac{1}{2}K_z D|) \\ \dfrac{K^2 - 4p_F^2 + 8mt_\perp(1 + |\cos\frac{1}{2}K_z D|)}{2(mt_\perp|\cos\frac{1}{2}K_z D|)^{1/2}} \\ \quad \text{for } 4p_F^2 - 8mt_\perp(1 + |\cos\tfrac{1}{2}K_z D|) < K^2 \\ \quad < 4p_F^2 - 8mt_\perp(1 - |\cos\tfrac{1}{2}K_z D|). \end{cases} \tag{12}$$

It is seen from (12) that the polarization operator $\Pi(K, K_z; 0)$ remains constant over a wide range of K, and there are corrections to $\Pi(0)$ only in the vicinity of $2p_F$. Substituting (12) and (8) into (9) we can calculate μ by using equation (7). For different asymptotic cases we obtain expressions for μ. In the case of $a > d$ the Coulomb pseudopotential is reduced to the following expression:

$$\mu(a, t_\perp) = 2\alpha/\pi\Big\{1/[2p_F a\alpha(2\epsilon_1 + 2p_F a\alpha)]^{1/2} + (1/\epsilon_1)\ln[(\alpha+\epsilon_1)/(\alpha+\epsilon_1/2p_F a)]$$

$$+ 1/(\epsilon_1+\alpha) + (4\alpha/\pi\epsilon_1)\sqrt{t_\perp/\epsilon_F}/(\epsilon_1+\alpha)\Big\}. \tag{13}$$

Here, the parameter $\tau = t_\perp/\epsilon_F$ characterizes the degree of anisotropy and $\tau < 1$ corresponds to an open Fermi surface. The other parameter $\alpha = e^2/\hbar v_F$ in (13) is defined as the ratio of the average Coulomb interaction between two electrons to the kinetic energy of an electron on the Fermi surface, and usually $\alpha \leqslant 1$.

In the opposite asymptotic case of $a < d$ equation (13) for $\mu(d, t_\perp)$ is obtained but here the replacements summarized as $\epsilon_1 \Leftrightarrow \epsilon$ and $a \Leftrightarrow d$ are employed.

Equation (13) shows that μ decreases as the thickness d of SC layers increases. As a consequence T_c increases. It can be seen from equation (13) that the dependence of T_c on the tunnelling integral $t_\perp$ has the following form:

$$T_c \propto \exp\left[-1\Big/\left(\lambda - \text{const.}\sqrt{t_\perp/\epsilon_F}\right)\right]. \tag{13'}$$

Since the superconducting properties of the system are defined by the electron–phonon coupling constant within the framework of the model under consideration, the dependence of λ on $t_\perp$, a and d should also be studied to find a complete expression for $T_c(t_\perp, a, d)$. On the other hand the essential order parameter phase fluctuations result in a rather strong dependence of T_c on $t_\perp$.

3. Influence of the order parameter phase fluctuations on the critical temperature: dependence of T_c on $t_\perp$ [24]

In purely 2D systems a correlator of order parameters $\langle\Delta_j(0)\Delta_j^*(r)\rangle$ is known to decrease as a power of the distance r. Thus, according to the Young criterion [9], ODLRO is absent in 2D superconductors. However, the existence of 'quasi-long-range order' in a system leads to the Kosterlitz–Thouless topological phase transition [11–13] at the critical temperature $T = T_{KT}$.

Further, the inclusion of even small coupling $t_\perp$ between SC layers restores an ODLRO in the system. Therefore T_c must increase with the transverse resonance integral $t_\perp$. Such a behaviour of T_c cannot be obtained from the BCS model and therefore the fluctuation effects must be taken into account.

It is necessary to emphasize that long-range order in a 2D SC is destroyed by strong fluctuation of the order parameter phase [9]. However, fluctuations of the order parameter modulus may be neglected. These features are also inherent to quasi-2D SCs [10].

To study the effect of order parameter phase fluctuations on the critical temperature T_c we shall start from the Lawrence–Doniach free energy functional $F_{st}\{\phi\}$ for a quasi-2D SC [6],

$$F_{st}\{\phi\} = N_s^{(2)} \sum_j \int d^2r \left\{ \frac{\hbar^2}{8m} \left[\left(\frac{\partial \phi_j}{\partial x} \right)^2 + \left(\frac{\partial \phi_j}{\partial y} \right)^2 \right] + \sum_{g=\pm 1} W_\perp(g)\{1 - \cos[\phi_j(x,y) - \phi_{j+g}(x,y)]\} \right\} \tag{14}$$

where $\phi_j(r)$ is the phase of the order parameter $\Delta_j = |\Delta_j| \exp[i\phi_j(r)]$ in the plane j with coordinate $r = \{x, y\}$. $W_\perp = t_\perp^2/\epsilon_F$ is the Josephson junction energy and $N_s^{(2)}(T)$ is the surface concentration of SC electrons defined as

$$N_s^{(2)}(T) = N_s^{(2)}(0)(1 - T/T_{c0}^{(2)}) \qquad \text{for } T < T_{c0}^{(2)}$$

where

$$N_s^{(2)}(0) = \tfrac{1}{2}(P_F/\hbar)^2$$

is the surface concentration of the normal state electrons.

In expression (14) contributions of the modulus of the order parameter $|\Delta_j|$ to $F_{st}\{\phi\}$ are neglected.

The mean value of the order parameter is defined by the following expression:

$$\langle \cos \phi_j \rangle = \int D\phi \cos \phi_j \exp\left(-\frac{F_{st}\{\phi\}}{kT} \right) \Big/ \int D\phi \exp\left(-\frac{F_{st}\{\phi\}}{kT} \right). \tag{15}$$

Accurate calculation of a path integral (15) with the free energy functional (14) is not possible. At $T = T_c$ equation (15) has a non-zero solution which may define the transition temperature T_c. To calculate the integral (15) we shall use the mean field approximation by replacing the free energy functional (14) by the following expression $\tilde{F}\{\phi\}$ [25]:

$$\tilde{F}\{\phi\} = N_s^{(2)} \int d^2r \left[\frac{\hbar^2}{8m} \left(\frac{\partial \phi(r)}{\partial r} \right)^2 - W_\perp \langle \cos \phi \rangle \cos \phi \right] = F_0\{\phi\} - N_s^{(2)} W_\perp \langle \cos \phi \rangle \int d^2r \cos \phi(r) \tag{16}$$

where $F_0\{\phi\}$ is the free energy functional of a 2D SC and $W_\perp = \sum_{g=\pm 1} W_\perp(g)$. This self-consistent method was used by Efetov and Larkin [25] to find the dependence of critical temperature T_c on $t_\perp$ in a quasi-one-dimensional SC (see also [26]).

In the vicinity of T_c the order parameter approaches zero. Therefore the second part of the free energy functional $\tilde{F}\{\phi\}$ in (16) may be chosen as a small parameter. Substituting (16) for $\tilde{F}\{\phi\}$ in (15) instead of $F_{st}\{\phi\}$ and performing an expansion in terms of this small parameter we obtain the following equation for T_c:

$$1 = \frac{N_s^{(2)}(T_c) W_\perp}{kT_c} \int d^2r \langle \cos \phi(0) \cos \phi(r) \rangle_0 \tag{17}$$

where the symbol $\langle \ldots \rangle_0$ indicates an averaging by means of the free energy functional $F_0\{\phi\}$ for a single SC layer. The correlator $\langle \cos\phi(0)\cos\phi(r)\rangle_0$ for a 2D SC has been calculated in [8,9]. It has the following form:

$$\langle \cos\phi(0)\cos\phi(r)\rangle_0 = \begin{cases} (\xi_\parallel/r)^{(1/\pi)kT/\epsilon_F(1-T/T_{c0}^{(2)})} & \text{for } r > \xi_\parallel \\ \exp\{-(1/\pi)[kT/\epsilon_F(1-T/T_{c0}^{(2)})](r/\xi_\parallel)^2\} & \text{for } r < \xi_\parallel \end{cases} \tag{18}$$

where $\xi_\parallel = \hbar v_F/\pi\Delta(0)$ is the correlation length inside the SC plane.

Equation (17) may be solved for T_c by taking into account the correlator expression (18) only when the following condition is satisfied:

$$(1/2\pi)kT_c/\epsilon_F(1 - T_c/T_{c0}^{(2)}) < 1. \tag{19}$$

In other words, the critical temperature must be in the interval of

$$T_c^* < T_c < T_{c0}^{(2)} \tag{20}$$

where T_c^* is defined as

$$1/kT_c^* = 1/kT_{c0}^{(2)} + 1/2\pi\epsilon_F. \tag{21}$$

T_c^* is the temperature above which the interlayer phase coherence is destroyed [8].

Substituting (18) into (17) under the condition (19) we obtain the following expression for the critical temperature T_c:

$$1/T_c = 1/T_{c0}^{(2)} - [1/2\pi\xi_\parallel^2 N^{(2)}(0)W_\perp][1 - (1 + 2\xi_\parallel^2 N^{(2)}(0)W_\perp/\epsilon_F)^{1/2}]. \tag{22}$$

For a small value of the tunnelling integral $W_\perp < \hbar^2/2m\xi_\parallel^2$, equation (22) takes the following form:

$$T_c = T_c^*/[1 - (T_c^*/2\pi)(2m\xi_\parallel^2/\hbar^2)(t_\perp/\epsilon_F)^2]. \tag{23}$$

For the opposite case, that is, $W_\perp > \hbar^2/2m\xi_\parallel^2$, the critical temperature becomes closer to $T_{c0}^{(2)}$, as can be seen from the relation

$$T_c = T_{c0}^{(2)}/(1 + T_{c0}^{(2)}/\pi k_F\xi_\parallel t_\perp). \tag{24}$$

It can be seen from equations (22)–(24) that the critical temperature T_c increases with increasing $t_\perp$ and approaches $T_{c0}^{(2)}$ in the interval $T_c^* < T_c < T_{c0}^{(2)}$.

It is necessary to emphasize that the above calculation of the Coulomb effect ((5), (6) and (13)) is carried out in the frame of BSC-like mean field theory. On the other hand quantum fluctuations of the phase of the order parameter also occur as a result of the electrostatic charging energy [27–30]. This effect has been studied earlier [8] for calculation of the correlator $\langle \cos\phi_j(0)\cos\phi_j(r)\rangle_0$ in quasi-2D SCs. In this case the Coulomb energy is characterized by the parameter

$$\alpha_0^2 = (\pi/2\gamma)^2 m/2\hbar^2 C$$

where **C** is the capacitance matrix. For small values of α_0^2, i.e. $\alpha_0^2 < 1$, the leading term of the correlator $\langle \cos\phi_j(0)\cos\phi_j(r)\rangle 0$ does not depend on the parameter α_0^2 (see [8]) and can be expressed by (18). Therefore, in the limit of $\alpha_0^2 < 1$ quantum fluctuations must not strongly change the classical fluctuation effects.

If we carry out calculations beyond the leading term in $\alpha_0^2 < 1$, then the correlator given in (18) must be multiplied by $\exp[-2\alpha_0^2(T_{c0}^{(2)}/T)kT_{c0}^{(2)}/\epsilon_F] < 1$.

The appearance of such a multiplication factor of the correlator in (18) is equivalent to renormalizing the Josephson coupling constant $W_\perp$ in (17). Further, this small exponential factor will reduce the value of $W_\perp$. Thus, the Coulomb energy resulting from the quantum fluctuations of the phase of the order parameter must lead to a reduction in T_c [27–30].

The charging effects and superconductor–insulator transition [27–30] in layered SCs are presently under investigation.

4. Conclusions

In this article the influence of the Coulomb repulsion and order parameter phase fluctuations on T_c has been studied in layered SCs. In the calculation of the Coulomb pseudopotential the thicknesses of the SC layers are also taken into consideration. By using equation (8) for a 'bare' Coulomb potential, the dependence of μ on $t_\perp$, a and d (see (13)) has been obtained.

Recently other theories [26–29] have appeared that provide different explanations of the dependence of T_c on $t_\perp$ and the number of CuO_2 layers in the unit cell for the high-T_c SC. To obtain the dependence of T_c on the number $n(n > 1)$ of SC layers in each elemental cell (or on the thickness of SC layers) a generalized Ginzburg–Landau functional with different critical temperature for each SC layer is used [28]. In this model the critical temperature T_c was shown to increase with increasing n. Our expression (13) for $\mu(d)$ reveals only the Coulomb repulsion contribution to T_c.

We must point out that the dependence of T_c on $t_\perp$ given in formula (13′) is not complete. To obtain the complete relation between T_c and $t_\perp$, a and d within Eliashberg's approach, the dependences of the effective electron–phonon coupling constant $\alpha^2(\omega)$ and the phonon density of states $F(\omega)$ on $t_\perp$, a and d must be studied. Though similar problems have been dealt with in the literature [30,31], we believe that the dependence of the electron–phonon coupling constant

$$\lambda = 2\int_0^\infty \frac{\alpha^2(\omega)F(\omega)}{\omega}\mathrm{d}\omega$$

on $t_\perp$, a and d should be investigated for a quasi-2D SC.

By taking into account the influence of order parameter phase fluctuations on the critical temperature we have shown that T_c decreases with $t_\perp$ (see (22)–(24)) within the interval $T_c^* < T_c < T_c^{(2)}$, approaching T_c^*. This indicates destruction of the coupling between the SC planes. However, the existence of vortices inside layers gives rise to a non-zero value of T_c.

In artificial $YBa_2Cu_3O_7/PrBa_2Cu_3O_7$ superlattices [5,6] the dependence of T_c on SC and dielectric layer thicknesses was investigated. Increase of the dielectric layer thickness a (i.e. the decrease of $t_\perp$) leads to a decrease in T_c and also an increase in

the width of the resistive transition. To understand such a dependence of T_c on a, one should probably start from equation (22). Widening of the resistive transition [1–3] may be caused by the motion of free vortices in SC layers [11–13].

It is possible to study the concentration dependence of the critical temperature in high-T_c SCs [32–39] by the equations ((22)–(24)) that we have obtained for T_c.

To gain more information on the $t_\perp$ dependence of T_c experimental studies of SC transitions under external pressure may be helpful.

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